## Algorithms for NLP



## Speech Inference

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## Project Announcements

- Due date postponed: now due Sat 9/23 at 11:59pm
- Will be using Canvas for jar and write-up submission
- We will test as soon as this is set up
- Invites will be sent to everyone (will announce)
- Extra jar submission of your best system
- No spot-checks for extra jar... feel free to use approximations
- Instructions for submission will be added to website
- If using open-address w/ long keys, try this hash:
- int hash = ((int) (key ^ (key >>> 32)) * 3875239);


## Project Grading

- Late days: 5 total, use whenever
- But no credit for late submissions when you run out of late days!
- (Be careful!)
- Grading: Projects out of 10
- 6 Points: Successfully implemented what we asked
- 2 Points: Submitted a reasonable write-up
- 1 Point: Write-up is written clearly
- 1 Point: Substantially exceeded minimum metrics
- Extra Credit: Did non-trivial extension to project


## Source / Filter

- Articulation process:
- The vocal cord vibrations create harmonics
- The mouth is an amplifier
- Depending on shape of mouth, some harmonics are amplified more than others


Deconvolution / Liftering


Deconvolution / Liftering


## Deconvolution / Liftering

## $S$ 



## Deconvolution / Liftering

$$
\log (S)=\log (e)+\log (f)
$$

## Mel Freq. Cepstral Coefficients

- Do FFT to get spectral information
- Like the spectrogram we saw earlier
- Apply Mel scaling (New)
- Models human ear; more sensitivity in lower freqs
- Approx linear below 1 kHz , log above, equal samples above and below 1 kHz
- Take Log
- Do discrete cosine transform


## Final Feature Vector

- 39 (real) features per 10 ms frame:
- 12 MFCC features
- 12 delta MFCC features
- 12 delta-delta MFCC features
- 1 (log) frame energy
- 1 delta (log) frame energy
- 1 delta-delta (log frame energy)
- So each frame is represented by a 39D vector


## Acoustic Model

## Speech Model



## Acoustic Model



## HMMs for Continuous Observations

- Before: discrete set of observations
- Now: feature vectors are real-valued
- Solution 1: discretization
- Solution 2: continuous emissions
- Gaussians
- Multivariate Gaussians
- Mixtures of multivariate Gaussians
- A state is progressively
- Context independent subphone (~3 per phone)
- Context dependent phone (triphones)
- State tying of CD phone



## Vector Quantization

- Idea: discretization
- Map MFCC vectors onto discrete symbols
- Compute probabilities just by counting
- This is called vector quantization or VQ
- Not used for ASR any more
- But: useful to consider as a starting point

Codebook of 256


## Gaussian Emissions

- VQ is insufficient for topquality ASR
- Hard to cover highdimensional space with codebook
- Moves ambiguity from the model to the preprocessing
- Instead: assume the possible values of the observation vectors are normally distributed.
- Represent the observation likelihood function as a Gaussian?


## Gaussians for Acoustic Modeling

A Gaussian is parameterized by a mean and a variance:

$$
P(x \mid \mu, \sigma)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left(-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right)
$$

- $P(x)$ :



## Multivariate Gaussians

- Instead of a single mean $\mu$ and variance $\sigma^{2}$ :

$$
P(x \mid \mu, \sigma)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left(-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right)
$$

- Vector of means $\mu$ and covariance matrix $\Sigma$

$$
P(x \mid \mu, \Sigma)=\frac{1}{(2 \pi)^{k / 2}|\Sigma|^{1 / 2}} \exp \left(-\frac{1}{2}(x-\mu)^{\top} \Sigma^{-1}(x-\mu)\right)
$$

- Usually assume diagonal covariance (!)
- This isn't very true for FFT features, but is less bad for MFCC features


## Gaussians: Size of $\Sigma$





- $\mu=[00]$

$$
\mu=\left[\begin{array}{ll}
0 & 0
\end{array}\right]
$$

$$
\mu=\left[\begin{array}{ll}
0 & 0
\end{array}\right]
$$

- $\Sigma=1$
$\Sigma=0.61$
$\Sigma=21$
- As $\Sigma$ becomes larger, Gaussian becomes more spread out; as $\Sigma$ becomes smaller, Gaussian more compressed


## Gaussians: Shape of $\Sigma$




$$
\Sigma=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] ; \quad \Sigma=\left[\begin{array}{cc}
1 & 0.5 \\
0.5 & 1
\end{array}\right] ; . \Sigma=\left[\begin{array}{cc}
1 & 0.8 \\
0.8 & 1
\end{array}\right]
$$

- As we increase the off diagonal entries, more correlation between value of $x$ and value of $y$


## But we're not there yet

- Single Gaussians may do a bad job of modeling a complex distribution in any dimension
- Even worse for diagonal covariances
- Solution: mixtures of Gaussians


From openlearn.open.ac.uk

## Mixtures of Gaussians

- Mixtures of Gaussians:

$$
\begin{aligned}
& P\left(x \mid \mu_{i}, \Sigma_{i}\right)=\frac{1}{(2 \pi)^{k / 2}\left|\Sigma_{i}\right|^{1 / 2}} \exp \left(-\frac{1}{2}\left(x-\mu_{i}\right)^{\top} \Sigma_{i}^{-1}\left(x-\mu_{i}\right)\right) \\
& P(x \mid \mu, \boldsymbol{\Sigma}, \mathbf{c})=\sum_{i} c_{i} P\left(x \mid \mu_{i}, \Sigma_{i}\right)
\end{aligned}
$$



From robots.ox.ac.uk



## GMMs

- Summary: each state has an emission distribution $\mathrm{P}(\mathrm{x} \mid \mathrm{s})$ (likelihood function) parameterized by:
- M mixture weights
- M mean vectors of dimensionality D
- Either M covariance matrices of DxD or M Dx1 diagonal variance vectors

- Like soft vector quantization after all
- Think of the mixture means as being learned codebook entries
- Think of the Gaussian densities as a learned codebook distance function
- Think of the mixture of Gaussians like a multinomial over codes
- (Even more true given shared Gaussian
 inventories... more soon)


## State Model

## State Transition Diagrams

- Bayes Net: HMM as a Graphical Model

- State Transition Diagram: Markov Model as a Weighted FSA



## ASR Lexicon



Figure: J \& M

## Lexical State Structure



Figure: J \& M

## Adding an LM



Figure from Huang et al page 618

## State Space

- State space must include
- Current word (|V| on order of 20K+)
- Index within current word (|L| on order of 5)
- E.g. (lec[t]ure) (though not in orthography!)
- Acoustic probabilities only depend on phone type
- E.g. $\mathrm{P}(\mathrm{x} \mid \operatorname{lec}[\mathrm{t}] \mathrm{ure})=\mathrm{P}(\mathrm{x} \mid \mathrm{t})$
- From a state sequence, can read a word sequence


## State Refinement

## E Phones Aren't Homogeneous



## Need to Use Subphones



Figure: J \& M

## E A Word with Subphones



Figure: J \& M

## $\pm$ <br> Modeling phonetic context



## "Need" with triphone models



Figure: J \& M

## Lots of Triphones

- Possible triphones: 50x50x50=125,000
- How many triphone types actually occur?
- 20K word WSJ Task (from Bryan Pellom)
- Word internal models: need 14,300 triphones
- Cross word models: need 54,400 triphones
- Need to generalize models, tie triphones


## State Tying / Clustering

- [Young, Odell, Woodland 1994]
- How do we decide which triphones to cluster together?
- Use phonetic features (or 'broad phonetic classes')
- Stop
- Nasal
- Fricative
- Sibilant
- Vowel
- lateral


Figure: J \& M

## State Space

- State space now includes
- Current word: $|\mathrm{W}|$ is order 20K
- Index in current word: |L| is order 5
- Subphone position: 3
- E.g. (lec[t-mid]ure)
- Acoustic model depends on clustered phone context
- But this doesn't grow the state space
- But, adding the LM context for trigram+ does
- (after the, lec[t-mid]ure)
- This is a real problem for decoding


## Decoding

## Inference Tasks



Most likely word sequence:

$$
d \quad-\quad \text { ae } \quad-\quad d
$$

Most likely state sequence:

$$
d_{1}-d_{6}-d_{6}-d_{4}-\mathrm{ae}_{5}-\mathrm{ae}_{2}-\mathrm{ae}_{3}-\mathrm{ae}_{0}-\mathrm{d}_{2}-\mathrm{d}_{2}-\mathrm{d}_{3}-\mathrm{d}_{7}-\mathrm{d}_{5}
$$

## ${ }_{x}$ <br> Viterbi Decoding



$$
\begin{gathered}
\phi_{t}\left(s_{t}, s_{t-1}\right)=P\left(x_{t} \mid s_{t}\right) P\left(s_{t} \mid s_{t-1}\right) \\
v_{t}\left(s_{t}\right)=\max _{s_{t-1}} \phi_{t}\left(s_{t}, s_{t-1}\right) v_{t-1}\left(s_{t-1}\right)
\end{gathered}
$$

Figure: Enrique Benimeli

## Viterbi Decoding



Figure: Enrique Benimeli

## Emission Caching

- Problem: scoring all the $P(x \mid s)$ values is too slow
- Idea: many states share tied emission models, so cache them



## Prefix Trie Encodings

- Problem: many partial-word states are indistinguishable
- Solution: encode word production as a prefix trie (with pushed weights)

- A specific instance of minimizing weighted FSAs [Mohri, 94]


## Beam Search

- Problem: trellis is too big to compute v(s) vectors
- Idea: most states are terrible, keep v(s) only for top states at each time

- Important: still dynamic programming; collapse equiv states


## LM Factoring

- Problem: Higher-order n-grams explode the state space
- (One) Solution:
- Factor state space into (word index, Im history)
- Score unigram prefix costs while inside a word
- Subtract unigram cost and add trigram cost once word is complete



## LM Reweighting

- Noisy channel suggests

$$
P(x \mid w) P(w)
$$

- In practice, want to boost LM

$$
P(x \mid w) P(w)^{\alpha}
$$

- Also, good to have a "word bonus" to offset LM costs

$$
P(x \mid w) P(w)^{\alpha}|w|^{\beta}
$$

- These are both consequences of broken independence assumptions in the model


## Training

## What Needs to be Learned?



- Emissions: P(x|phone class)
- X is MFCC-valued
- Transitions: P(state | prev state)
- If between words, this is P(word | history)
- If inside words, this is P(advance \| phone class)
- (Really a hierarchical model)


## Estimation from Aligned Data

- What if each time step was labeled with its (contextdependent sub) phone?

- Can estimate $P(x \mid / a e /)$ as empirical mean and (co-)variance of $x^{\prime}$ s with label/ae/
- Problem: Don’t know alignment at the frame and phone level


## Forced Alignment

- What if the acoustic model P(x|phone) was known?
- ... and also the correct sequences of words / phones
- Can predict the best alignment of frames to phones
"speech lab"
ssssssssppppeeeeeeetshshshshllllaeaeaebbbbb

- Called "forced alignment"


## Forced Alignment

- Create a new state space that forces the hidden variables to transition through phones in the (known) order

- Still have uncertainty about durations
- In this HMM, all the parameters are known
- Transitions determined by known utterance
- Emissions assumed to be known
- Minor detail: self-loop probabilities
- Just run Viterbi (or approximations) to get the best alignment


## EM for Alignment

- Input: acoustic sequences with word-level transcriptions
- We don't know either the emission model or the frame alignments
- Expectation Maximization (Hard EM for now)
- Alternating optimization
- Impute completions for unlabeled variables (here, the states at each time step)
- Re-estimate model parameters (here, Gaussian means, variances, mixture ids)
- Repeat
- One of the earliest uses of EM!


## Soft EM

- Hard EM uses the best single completion
- Here, single best alignment
- Not always representative
- Certainly bad when your parameters are initialized and the alignments are all tied
- Uses the count of various configurations (e.g. how many tokens of /ae/ have self-loops)
- What we'd really like is to know the fraction of paths that include a given completion
- E.g. 0.32 of the paths align this frame to /p/, 0.21 align it to /ee/, etc.
- Formally want to know the expected count of configurations
- Key quantity: $P\left(s_{t} \mid x\right)$


## Computing Marginals



$$
\begin{gathered}
P\left(s_{t} \mid x\right)=\frac{P\left(s_{t}, x\right)}{P(x)} \\
=\frac{\text { sum of all paths through s at } t}{\text { sum of all paths }}
\end{gathered}
$$

## Forward Scores



$$
\begin{aligned}
& v_{t}\left(s_{t}\right)=\max _{s_{t-1}} v_{t-1}\left(s_{t-1}\right) \phi_{t}\left(s_{t-1}, s_{t}\right) \\
& \alpha_{t}\left(s_{t}\right)=\sum_{s_{t-1}} \alpha_{t-1}\left(s_{t-1}\right) \phi_{t}\left(s_{t-1}, s_{t}\right)
\end{aligned}
$$

## $\pm$ Backward Scores



$$
\beta_{t}\left(s_{t}\right)=\sum_{s_{t+1}} \beta_{t+1}\left(s_{t+1}\right) \phi_{t}\left(s_{t}, s_{t+1}\right)
$$

## Total Scores



$$
\begin{aligned}
& P\left(s_{t}, x\right)=\alpha_{t}\left(s_{t}\right) \beta_{t}\left(s_{t}\right) \\
& \begin{aligned}
P(x) & =\sum_{s_{t}} \alpha_{t}\left(s_{t}\right) \beta_{t}\left(s_{t}\right) \\
& =\alpha_{T}(\text { stop }) \\
& =\beta_{0}(\text { start })
\end{aligned}
\end{aligned}
$$

## Fractional Counts

- Computing fractional (expected) counts
- Compute forward / backward probabilities
- For each position, compute marginal posteriors
- Accumulate expectations
- Re-estimate parameters (e.g. means, variances, self-loop probabilities) from ratios of these expected counts


## Staged Training and State Tying

- Creating CD phones:
- Start with monophone, do EM training
- Clone Gaussians into triphones
- Build decision tree and cluster Gaussians
- Clone and train mixtures (GMMs)
- General idea:
- Introduce complexity gradually
- Interleave constraint with flexibility



## Training Mixture Models

- Input: wav files with unaligned transcriptions
- Forced alignment
- Computing the "Viterbi path" over the training data (where the transcription is known) is called "forced alignment"
- We know which word string to assign to each observation sequence.
- We just don't know the state sequence.
- So we constrain the path to go through the correct words (by using a special example-specific language model)
- And otherwise run the Viterbi algorithm
- Result: aligned state sequence


## State Tying

- Creating CD phones:
- Start with monophone, do EM training
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## Standard subphone/mixture HMM



| Model | Error rate |
| :--- | ---: |
| HMM Baseline | $25.1 \%$ |

## An Induced Model


[Petrov, Pauls, and Klein, 07]

## Hierarchical Split Training with EM



## Refinement of the /ih/-phone



## Refinement of the /ih/-phone



## Refinement of the /ih/-phone



## HMM states per phone



